A THEOREM FOR TWO NUCLEON TRANSFER ON RADIOACTIVE ^{44}Ti

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NATO COLLABORATION

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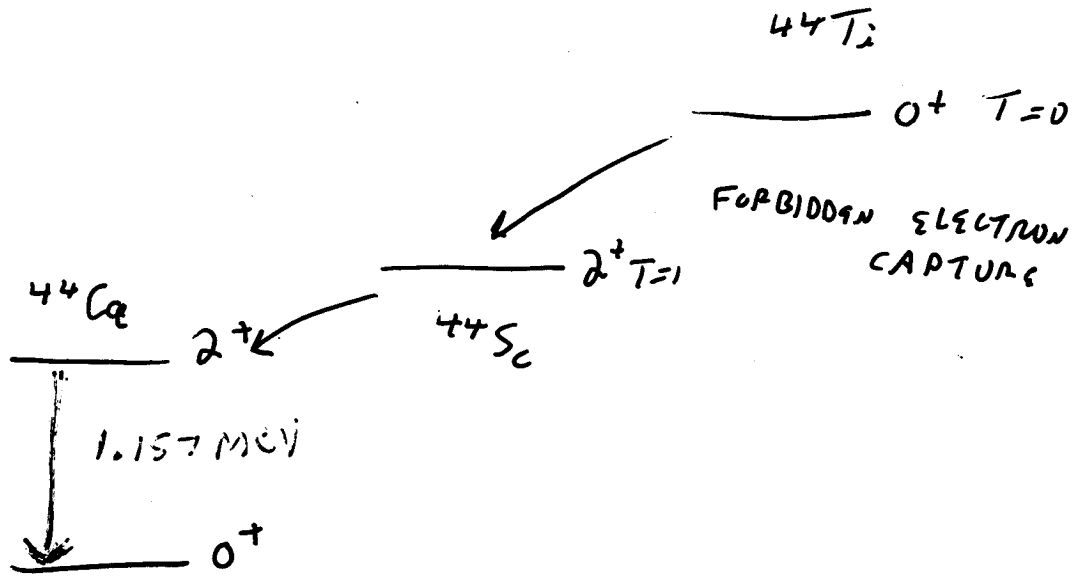
INITIAL MOTIVATION

$J=0$ $T=1$ pairing vs. $J=1$ $T=0$ pairing

NEW MOTIVATION

SURROGATES CAN LEAD TO SIMPLICITY



LIFETIME of ^{44}Ti 59.2 ± 0.6 years

BUT IS THE LIFETIME different in a SUPERNOVA EXPLOSION?

If all electrons were stripped there would be no electron capture

IF 1.157 MeV γ ray is seen then supernova explosion was relatively recent

e.g. CASSIOPEIA A ~ 365 years

ANATOMY OF A SHELL MODEL CALCULATION

$$[[j\,i]J_P\ [i\,i]J_N]I \qquad E(J) = \langle (j\,i)^J V (i\,i)^J]$$

$$\langle [J_P' J_N'] \quad V \quad [J_P J_N] \rangle$$

$$= [\underset{PP}{E(J_P)} + \underset{NN}{E(J_N)}]\ \delta_{J_P' J_P}\ \delta_{J_N' J_N}$$

$$+ 4 \sum_{\substack{J_A \\ J_B}} \langle (j\,i) J_P' (i\,i) J_N' | (j\,i) J_A\ (i\,i) J_B) I$$

$$\langle (j\,i) J_P (i\,i) J_N | (i\,i) J_A\ (i\,i) J_B) I$$

$$\underset{PN}{E(J_B)}$$

# of PAIRS of ANGULAR MOMENTUM J_{12}

$$\psi = \sum D^I(J_P J_N)\ [J_P J_N]^I$$

$$\langle \psi | H \psi \rangle = \sum_{J_{12}} \#\text{ of PAIRS of ANG MOMENTUM } (J_{12})\ E(J_{12})$$

^{44}Ti

E (MeV)

	0	5.586	8.284	8.788
D(00)	0.788	−0.352	−0.500	−0.072
D(22)	0.562	0.737	0.373	−0.050
D(44)	0.221	−0.370	0.500	0.751
D(66)	0.123	−0.442	0.601	−0.654
	T=0	T=0	T=2	T=0

SHARON

SPEIDEL - BONN

^{44}Ti $\quad g(2^+) = 0.52(15)$

IN GENERAL

$$g = \left(\frac{g_\pi + g_\nu}{2}\right) + \sum_{J_P J_N} |D(J_P J_N)|^2 \left[J_P(J_P+1) - J_N(J_N+1)\right] \times \left(\frac{g_\pi - g_\nu}{2} \frac{}{I(I+1)}\right)$$

FOR ^{44}Ti

Seiler: $g = \left(\frac{g_\pi + g_\nu}{2}\right) = 0.554$ ISOSCALAR !!

Collective $g = Z/A = 0.5$

$(fp)^4$ $\quad g = 0.532$

Augusto Macciavelli $\quad ^{44}Ti(^3He,p)^{46}V$

3.

6 SPECTRUM of ^{42}Sc

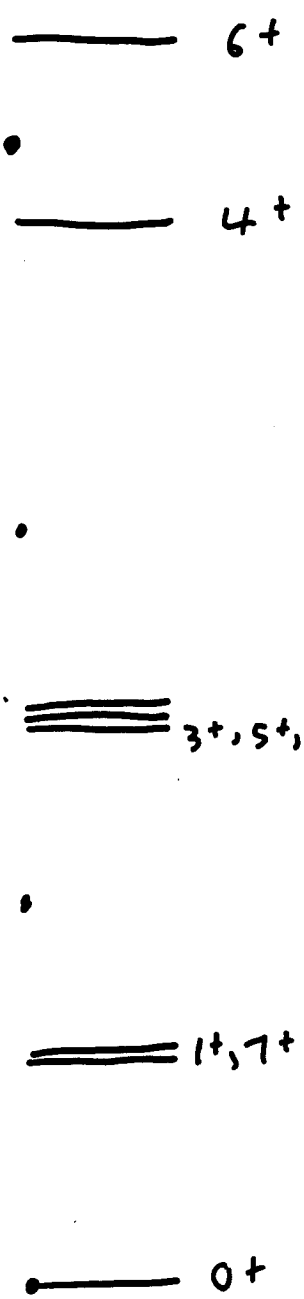
MeV

4 •

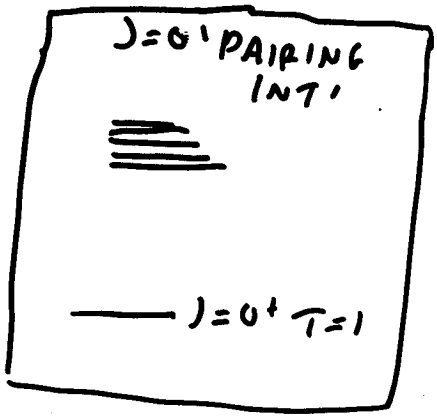
3 •

2 •

1 •



a)



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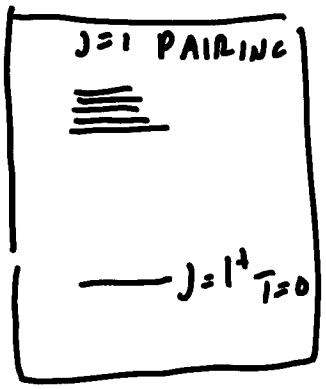

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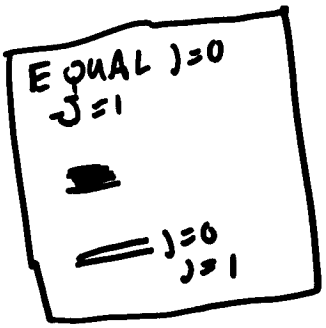


EVEN J T=1 !

ODD J T=0 !

$$\Psi^I = \sum_{J_P J_N} D^I(J_P, J_N) \left[(j^2)^{J_P} (j^2)^{J_N} \right]^{\pm}$$

$|D^I(J_P J_N)|^2 \equiv$ PROBABILITY THAT IN A STATE of TOTAL ANGULAR MOMENTUM I PROTONS couple to J_P and neutrons to J_N.

For $I=0$ $\quad J_P = J_N$

of J_{12} PAIRS =

$$2\,|D(J_{12}, J_{12})|^2 + |f(J_{12})|^2$$

$$f(J_{12}) = 2\,(2J_{12}+1) \sum_{\substack{J \\ \text{EVEN}}} (2J+1)\, D(J,J) \begin{Bmatrix} j & j & J \\ j & j & J \\ J_{12} & J_{12} & 0 \end{Bmatrix}$$

FOR A SYSTEM of n nucleons in a single j shell with total isospin T we have

TOTAL NUMBER of PAIRS $\frac{n(n-1)}{2}$

$$\boxed{\text{Total number of } T_{12}=0 \text{ pairs} = \frac{n^2}{8} + \frac{n}{4} - \frac{T(T+1)}{2}}$$

$$\boxed{\text{Total number of } T_{12}=1 \text{ pairs} \quad \frac{3n^2}{8} - \frac{3n}{4} + \frac{T(T+1)}{2}}$$

THIS IS INDEPENDANT of the INTERACTION !

e.g. $^{44}T_i$ $\quad \frac{n(n-1)}{2} = \frac{4\times 3}{2} = 6$ PAIRS

3 $T_{12}=0$ pairs

3 $T_{12}=1$ pairs

6. 9

INTERACTIONS	NUMBER OF PAIRS $J_{12}=0^+$, $T_{12}=1$	$J_{12}=1^+$, $T_{12}=0$	$J_{12}=7^+$, $T_{12}=0$
(1) $J=0$, $T=1$ PAIRING — $J=0^+$, $T=1$	2.250	0.250	1.250
(2) $J=1$, $T=0$ PAIRING — $J=1^+$, $T=0$	0.433	1.297	1.311
(3) EQUAL $J=0$ and $J=1$ — $1^+, 0^+$	2.045	0.618	1.654
(4) REALISTIC — 6, 4, 2, 0, 3, 5, 1, 3, 7, 0	1.736	0.746	1.948

^{44}Ti $J=0^+$ $T=0$

of J_{12} pairs

(SPECTRUM of ^{42}Sc)

NO INTERACTION

J_{12}		
0	1.736	0.750
1	0.746	0.432
2	1.126	0.861
3	0.216	0.902
4	0.114	0.750
5	0.091	1.000
6	0.020	0.639
7	1.948	0.667

^{44}Ca $(T=2 \text{ in } ^{44}Ti)$

$J=0$ SENIORITY ZERO $(f_{7/2}^{4})$

# of PAIRS

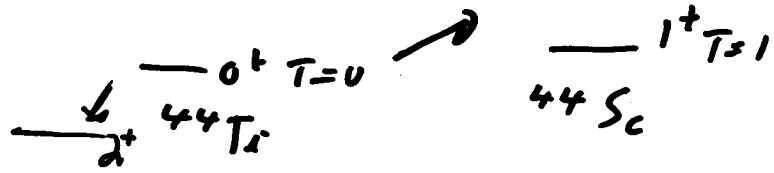
$J_{12}=0$	1.5
2	0.8333
4	1.5
6	2.16667

MORE $J=6$ pairs than $J=0$ pairs

^{44}Ca (HH) $0^+ \to I$ $\propto (2I+1)$

THE MORE SPHERICAL A SYSTEM IS THE MORE HIGH ANGULAR MOMENTUM PAIRS THERE ARE.

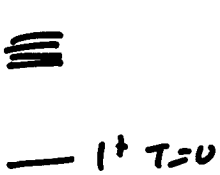
GAMOW Teller



J=0 T=1
PAIRING

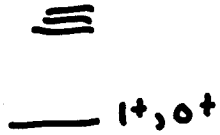
B(GT) = 1.14

J=1 T=0
PAIRING



B(GT) = 2.46

EQUAL



B(GT) = 0.73

McCullen
Interaction

B(GT) = 0.55

NEW RELATIONSHIP FOR TWO NUCLEON TRANSFER

$$U_{qj}(J_A, J_B) \equiv \langle (jj)J_A (jj)J_A | (jj)J_B (jj)J_B \rangle_0$$

$$\boxed{\begin{aligned} \#\text{ of PAIRS} &= 2\,|D(J_{12}, J_{12})|^2 + |f(J_{12})|^2 \\ f(J_{12}) &= 2 \sum_{J_P} U_{qj}(J_P, J_{12})\, D(\bar{J}_P \bar{J}_P) \end{aligned}}$$

NEW RELATIONSHIPS → for J_{12} even!

$$\sum_{J_P} U_{qj}(J_P J_{12})\, D(\bar{J}_P \bar{J}_P) = D(J_{12} J_{12})/2 \qquad T=0$$

$$= -D(J_{12} J_{12}) \qquad T=2$$

$$\boxed{\text{HENCE } \#\text{ of (even) } J_{12} \text{ pairs} = 3\,|D(J_{12} J_{12})|^2}$$

PROOF: Consider the square matrix "HAMILTONIAN"

$\boxed{U_{qj}(J_P, J_{12})}$ for even J_P and even J_{12}

$J_P = 0, 2, 4, 6$

$J_{12} = 0, 2, 4, 6$

DIAGONALIZE THIS qj

Eigenvalues are $\frac{1}{2}$ (TRIPLY degenerate) $(T=0)$

-1 singly degenerate $(T=2)$

DEEPER EXPLANATION

NP2

The Interrelationship of angular momentum and isospin

Consider the simple 'nucleon-nucleon' interaction

$$V = \sum_{i<j} \left[\frac{1}{4} - t(i)\cdot t(j) \right]$$

$$= \frac{1}{4}\frac{(m)(m-1)}{2} - \frac{1}{2}\sum_{i,j} t(i)\cdot t(j) + \frac{1}{2}\sum_i t(i)^2$$

$$= \frac{m(m-1)}{8} - \frac{1}{2}T(T+1) + 3/8\, m$$

$$= \boxed{\frac{m^2}{8} + \frac{m}{4} - \frac{1}{2}T(T+1)}$$

For $m=4$ $T=0$ answer is 3

$T=2$ " " 0

BUT WE CAN GET THIS RESULT IN A DUMB WAY

IN ANGULAR MOMENTUM SPACE $V = \dfrac{(1-(-1)^{J_{12}})}{2}$

15

NR3

$$V = \left[\frac{1}{4} - t(1)\cdot t(2)\right] = \frac{\left(1-(-1)^{J_{12}}\right)}{2}$$

$$\langle [J_p' J_p]^0 \; V \; [J_p J_p]^0 \rangle = 2 + 2U9j(J_p', J_p)$$

EIGENVALUES MUST BE

$$\frac{n^2}{8} + \frac{n}{4} - \frac{1}{2}(T)(T+1)$$

$\left.\begin{matrix} n=4 \\ T=0 \end{matrix}\right\} 3$ $\qquad$ $\left.\begin{matrix} n=4 \\ T=2 \end{matrix}\right\} 0$

$2 + 2U9j = 3$ $\qquad$ $2 + 2U9j = 0$

$U9j = 1/2$ $\qquad$ $U9j = -1$

CONCLUSIONS

① SOME RADIOACTIVE NUCLEI are SIMPLER THAN STABLE ONES

^{44}Ti vs. ^{46}Ti

② FOR 2 NUCLEON PICKUP from ^{44}Ti the effect of the interaction is to <u>enhance</u> the number of $J=0^+, 1^+, 2^+$ and 7^+

and to <u>decrease</u> the number of $J=3^+, 4^+, 5^+$ and 6^+ states

③ GAMOW TELLER IS BEST INDICATION of $J=0^+$ vs $J=1^+$ pairing

ONLY $J=0$	$B(GT)=1.14$
$J=1$	2.46
$J=0+1$	0.73

④ NEW RELATION $\sum UCJ(J_A, J_B)\, D(J_B, J_B) = \dfrac{D(J_A, J_A)}{2}$

BRING ON

THE

SURROGATES